Quiz 2 - Friday 9/8 - Lessons 4-6.
Bring Blue or Black Pen

Lesson 7 - Integration By Parts - Part I
I. Another Tool for integrating products
II. Examples

## I. Another Tool for integrating products

1st Tool - u-substitution, from chain rule
2nd tool - "by parts", from product rule

$\boxed{Ex}$ $\int x \ln(x^5)\, dx$

Last class

$u = \ln(x^5)$

$du = \frac{1}{x^5} \cdot 5x^4\, dx$

$du = \frac{1}{x}\, dx$

missing - This won't work

What if we try

$u = x^5$

$du = 5x^4\, dx$

$\frac{1}{5} du = x^4 dx$

missing - This won't work

New technique today. Derived from product rule.

$$\frac{d}{dx}[uv] = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\Rightarrow \int \left[u\frac{dv}{dx} + v\frac{du}{dx}\right] dx = uv + C$$

$$\Rightarrow \int u \frac{dv}{dx}\, dx + \int v \frac{du}{dx}\, dx = uv + C$$

$$\Rightarrow \int u\, dv + \int v\, du = uv + C$$

$$\Rightarrow \int u\, dv = uv - \int v\, du + C$$

Integration By Parts Formula

$$\boxed{\int u\, dv = uv - \int v\, du}$$

Back to our example . . . .

$$\int x \underbrace{\ln(x^5)}_{u}\, dx \quad \text{dv}$$

| | |
|---|---|
| $u = \ln(x^5)$ | $dv = x\,dx$ |
| $du = \frac{1}{x^5} \cdot 5x^4$ | $\int dv = \int x\,dx$ |
| $du = \frac{5}{x}\, dx$ | $v = \frac{x^2}{2}$ |

in by parts, don't add C until last step

$$\int u\, dv = uv - \int v\, du$$

$$\boxed{\int x \ln(x^5)\, dx} = \ln(x^5) \cdot \frac{x^2}{2} - \int \frac{x^2}{2} \cdot \frac{5}{x}\, dx$$

$$= \frac{x^2}{2} \ln(x^5) - \int \frac{5}{2} x\, dx$$

$$= \frac{x^2}{2} \ln(x^5) - \frac{5}{2} \frac{x^2}{2} + C$$

$$= \boxed{\frac{x^2}{2} \ln(x^5) - \frac{5}{4} x^2 + C}$$

$$\int u\,dv = uv - \int v\,du$$

Lessons from this example

1. Whatever you choose for dv must be something you can integrate
2. $\int v\,du$ needs to be easier than $\int u\,dv$
3. Everything in the original integral must be in u or dv.

What should we choose for u? (Be default, this also chooses dv.)

Thanks to Alexandra Cuadra

Not foolproof, but often works.

Order for choosind u

1. L ogs
2. A lgebraic (Polynomials, Roots)
3. T rig (sin, cos)
4. E xponential ($e^x$, $e^{3x}$)

II. Examples

$\boxed{EX}$ $\int (2x+1)e^x\,dx = (2x+1)e^x - \int e^x 2dx$

$u = 2x+1$ $\quad dv = e^x dx$

$du = 2dx$ $\quad \int dv = \int e^x dx$

$v = e^x$

$= (2x+1)e^x - 2e^x + C$

$= (2x+1-2)\,e^x + C$

$= (2x-1)\,e^x + C$

$\boxed{EX}$ $\int x(x+5)^{10}\,dx$

3 options

1. Expand $x(x+5)^{10}$ and use power rule — yuck!

Easiest. → 2. $u = x+5 \Rightarrow x = u-5$ $\quad \int (u-5)u^{10}\,du = \int (u^{11} - 5u^{10})\,du$

$du = dx$

power rule.

③ By parts. $\int x(x+5)^{10}\,dx = (x+5)^{10}\cdot\frac{x^2}{2} - \underbrace{\int \frac{x^2}{2}\cdot 10(x+5)^9\,dx}_{\text{not better try again}}$

$\cancel{u = (x+5)^{10}}$ $\quad$ $dv = x\,dx$

$\cancel{du = 10(x+5)^9\cdot 1\,dx}$ $\quad$ $\int dv = \int x\,dx$

$v = \frac{x^2}{2}$

$u = x$ $\quad$ $dv = (x+5)^{10}\,dx$

$\boxed{du = dx}$ $\quad$ $\int dv = \int (x+5)^{10}\,dx$ $\quad$ $b = x+5$

$v = \int b^{10}\,db$ $\quad$ $db = dx$

$= \frac{b^{11}}{11}$

$\boxed{v = \frac{(x+5)^{11}}{11}}$

$$\int x(x+5)^{10}\,dx = \frac{x(x+5)^{11}}{11} - \int \frac{(x+5)^{11}}{11}\,dx$$

$$= \frac{x(x+5)^{11}}{11} - \frac{(x+5)^{12}}{11\cdot 12} + C$$

$\boxed{EX}$ $\int_0^{\pi/2} x\sin(2x)\,dx = x\cdot -\frac{1}{2}\cos(2x)\Big|_0^{\pi/2} - \int_0^{\pi/2} -\frac{1}{2}\cos(2x)\,dx$

$u = x$ $\quad$ $dv = \sin(2x)\,dx$ $\quad$ $x\cdot -\frac{1}{2}\cos(2x) + \frac{1}{2}\cdot\frac{1}{2}\sin(2x)\Big|_0^{\pi/2}$

$du = dx$ $\quad$ $\int dv = \int \sin(2x)\,dx$

$v = -\frac{1}{2}\cos(2x)$ $\quad$ Ans $\frac{\pi}{4}$